ME 202 - Strength of Materials

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DISCLAIMER

This document is a compilation of the notes I made while taking the course ME 202 (Strength of Materials) in my 4^{th} semester at IIT Bombay. Even though I try to discuss as much theory as possible, this is not a substitute to any formal teaching material on the subject.

There will probably be many instances where I use certain common symbols without explicitly mentioning what they mean. It is to be assumed that they carry their usual meanings.

If you have any suggestions and/or spot any errors, you know where to contact me.

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1 Torsion of Circular Shafts

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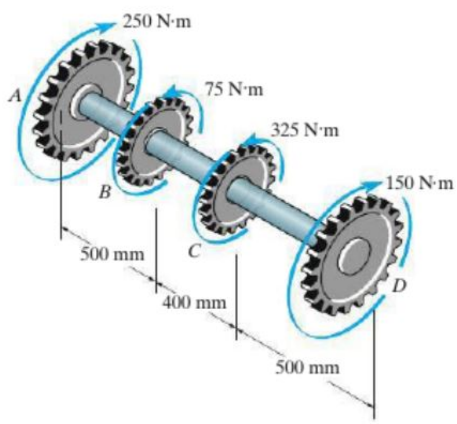
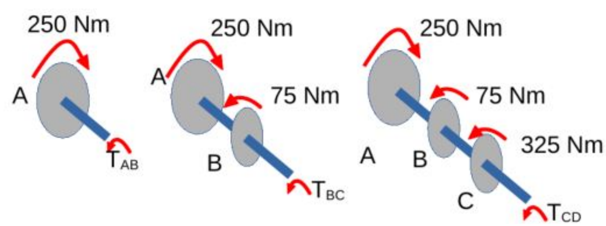
– torque: causes twist or *torsion* in a machine element

– shaft: transmits rotary motion from one location to another

i) Internal Resisting Torque (method of sections)



FBDs at different sections:

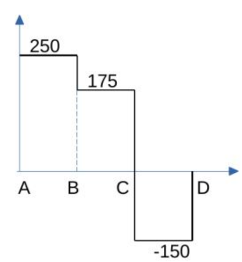


$$\sum M_{AB} = 0 \implies -250 + T_{AB} = 0$$

$$\sum M_{BC} = 0 \implies -250 + 75 + T_{BC} = 0$$

$$\sum M_{CD} = 0 \implies -250 + 75 + 325 + T_{CD} = 0$$

The results can be shown using a torque diagram.

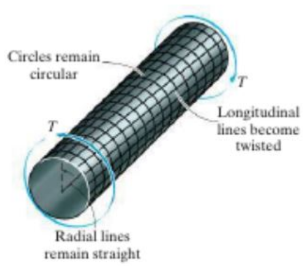


Direction of torque is decided using right hand thumb rule, i.e. thumb along +ve z-direction and direction of curling of fingers corresponds to +ve torque.

To define the origin, consider the element you come across first when you travel along that direction (A, in this case).

If external torque varies with z, take a section at an arbitrary z and find internal resisting torque as a function of z, i.e. $T(z)$.

ii) Some Observations



– each circular cut remains a circle

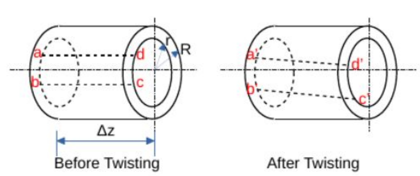
– longitudinal lines deform helically & intersect the circles at equal angles

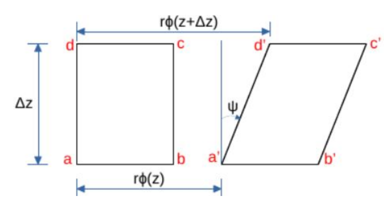
– cross-sections of shaft ends remain flat

– radial lines on the flat ends remain straight

Non-circular shafts often undergo a phenomenon called warping (a topic for a later time).

iii) Shear Strain γ



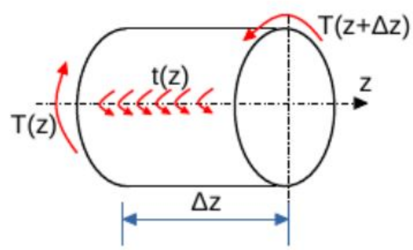


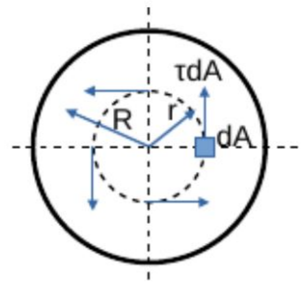
Notice how the element is initially rectangular and later gets distorted. This gives rise to a shear strain and as a result, shear stress. As a kinematic assumption, the cross-sections remain planar and rotate rigidly under the load.

$$\gamma = \tan\psi_{z\theta} = \frac{r\phi(z+\Delta z) - r\phi(z)}{\Delta z} \implies \boxed{\gamma = r\frac{\mathrm{d}\phi}{\mathrm{d}z}}$$

$$\therefore \gamma_{max} = R\frac{\mathrm{d}\phi}{\mathrm{d}z} \implies \gamma = \left(\frac{r}{R}\right)\gamma_{max}$$

iv) Shear Stress τ and the Torsion Formula





$$T(z+\Delta z) + t(z)\Delta z - T(z) = 0 \implies \frac{\mathrm{d}T}{\mathrm{d}z} + t(z) = 0$$

The moment due to the shear stresses must equal the external torque $T(z)$. Thus $T(z) = \int_{A(z)} \tau r \mathrm{d}A$.
Also since τ & γ satisfy the constitutive relation $\boxed{\tau = G\gamma}$, thus $\gamma = \left(\frac{r}{R}\right)\gamma_{max} \implies \tau = \left(\frac{r}{R}\right)\tau_{max}$.

$$\therefore T = \int_A \tau r \mathrm{d}A = \int_A \tau_{max}\frac{r^2}{R}\mathrm{d}A = \frac{\tau_{max}}{R}\int_A r^2 \mathrm{d}A \implies \boxed{\tau_{max} = \frac{TR}{J} \implies \tau = \frac{Tr}{J}}$$

The polar moment of inertia $J = \int_A r^2 \mathrm{d}A = \begin{cases} \dfrac{\pi R^4}{2} & \text{for solid cross-sections} \\ \dfrac{\pi(r_0^4 - r_i^4)}{2} & \text{for annular cross-sections} \end{cases}$

Due to the complementary properties of the shear stress, an associated shear stress is also developed in the plane parallel to the z-axis.

v) Angle of Twist