ME 202 - Tutorial 2 (08/02)

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I pledge on my honor that I have not sent my answer sheets to any of my course mates for the tutorial.

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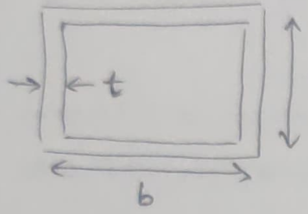


a) For a thin-walled shaft, since $t << 1$, it is okay to assume that $\tau_{avg} \approx \tau_{max}$.

$$\therefore \tau_{max} = \frac{T}{2tA_m} \implies \boxed{\tau_{max} = \frac{T}{2tab}}$$

b) We know that $p = 2(a+b)$ and $r = \frac{a}{b}$

$$\therefore a = rb \implies b = \frac{p}{2(r+1)} \quad \text{and} \quad a = \frac{pr}{2(r+1)}$$

$$\therefore \tau_{max} = \frac{T}{2 \times t \times \frac{pr}{2(r+1)} \times \frac{p}{2(r+1)}} \implies \boxed{\tau_{max} = \frac{2T(r+1)^2}{trp^2}}$$

c) Since p is fixed, for min. value of τ we must have $\frac{\partial \tau}{\partial r} = 0$

$$\therefore \frac{2T}{tp^2} \times \frac{\partial}{\partial r}\left[\frac{(r+1)^2}{r}\right] = 0$$

$$\therefore \frac{r \times 2(r+1) - (r+1)^2}{r^2} = 0 \implies \boxed{r = 1}$$

min. shear stress occurs for a square of a given perimeter

d) For a thin walled shaft under torsion, we have

$$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$$

since t is constant, $\therefore \oint \frac{ds}{t} = \frac{1}{t}\oint ds = \frac{p}{t}$

$$= \frac{TLp}{4A_m^2 G t}$$

$\therefore$ torsional stiffness $k_T = \frac{T}{\phi} \implies \boxed{k_T = \frac{4A_m^2 G t}{Lp}}$

e) From part (b), $b = \frac{p}{2(r+1)}$; $a = \frac{rp}{2(r+1)}$

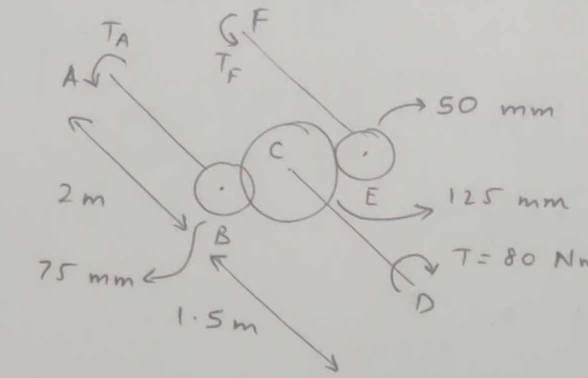
$$\therefore k_T = \frac{4 \times \frac{p^2}{4(r+1)^2} \times \frac{r^2p^2}{4(r+1)^2} \times Gt}{Lp} \implies \boxed{k_T = \frac{r^2p^3Gt}{4L(r+1)^4}}$$

f) Again for a fixed p, $\frac{\partial k_T}{\partial r} = 0$

$$\therefore \frac{\partial}{\partial r}\left[\frac{r^2}{(r+1)^4}\right] = 0 \Rightarrow \frac{(r+1)^4 \times 2r - r^2 \times 4(r+1)^3}{(r+1)^8} = 0$$

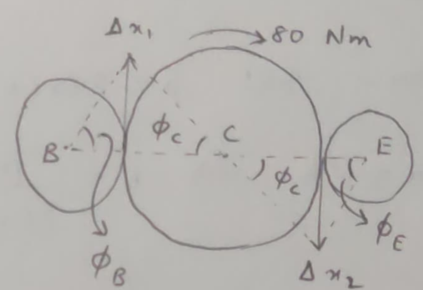
$$\therefore \boxed{r = 1}$$

(2) a) For simplicity, all shafts will be represented by solid lines and gears by circles in FBDs.

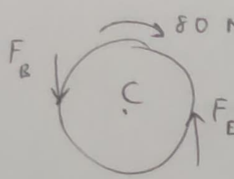


given shaft diameter $d = 20$ mm

shear modulus $G = 75$ GPa



(FBD showing tangential disp.)



(FBD of C)

From the FBD of C, we have

$$r_c (F_B + F_E) = 80$$

$$\therefore F_B + F_E = 640 \text{ N} \quad \text{—— (1)}$$

From geometric compatibility, the twist constraint relation gives us that $\Delta x_1 = \Delta x_2$. This is because the gears do not lose contact with each other during the operation.

$$\therefore \phi_B r_B = \phi_c r_c = \phi_E r_E \Rightarrow \phi_E = 1.5 \phi_B$$

$$\therefore \frac{T_E L_{EF}}{JG} = 1.5 \frac{T_B L_{AB}}{JG} \Rightarrow T_E = 1.5 T_B \quad \text{or} \quad r_E F_E = 1.5 r_B F_B$$

$$\therefore F_E = 2.25 F_B \quad \text{—— (2)}$$

$\therefore$ From (1) and (2), we get

$$F_B + 2.25 F_B = 640 \longrightarrow \therefore F_B = 194.9 \text{ N}; \; F_E = 433.1 \text{ N}$$

$$\therefore T_B = 14.77 \text{ Nm}; \; T_E = 22.15 \text{ Nm}$$

$$\therefore \boxed{\begin{aligned} T_A &= 14.77 \text{ Nm} \\ T_F &= 22.15 \text{ Nm} \\ T_{CD} &= 80 \text{ Nm} \end{aligned}}$$

b) $\tau = \frac{TR}{J} = \frac{2T}{\pi R^3}$

$\therefore \tau_{AB} = \frac{2 \times 14.77}{\pi \times (10^{-2})^3} \boxed{= 9.402 \text{ MPa}}$

$\tau_{EF} = \frac{2 \times 22.15}{\pi \times (10^{-2})^3} \boxed{= 14.101 \text{ MPa}}$

c) We can write : $\phi_{D/A} = \phi_{D/C} + \phi_{C/B} + \phi_{B/A}$

$= \phi_{D/C} + \phi_{C/A}$

To calculate $\phi_{C/A}$ (i.e. ϕ_c), we first find ϕ_B & then use $r_B \phi_B = r_c \phi_c$.

$\therefore \phi_B = \frac{14.77 \times 2}{75 \times 10^9 \times \frac{\pi}{2} (10^{-2})^4} = 0.025$ rad

$\therefore \phi_c = \frac{r_B \phi_B}{r_c} \Rightarrow \phi_c = 0.015$ rad (clockwise)

$\phi_{D/C} = \frac{80 \times 1.5}{75 \times 10^9 \times \frac{\pi}{2} (10^{-2})^4} \Rightarrow \phi_{D/C} = 0.102$ rad (clockwise)

$\therefore \phi_{D/A} = -0.102 - 0.015 \Rightarrow \boxed{\phi_{D/A} = -0.117 \text{ rad}}$